Cambridge Educational Research e-Journal

ISSN: 2634-9876 Journal homepage: http://cerj.educ.cam.ac.uk/

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To cite this entry:

Kassim-Lowe, T. (2024). Midas syndrome? Commognition as a lens for research on how stem sentences feature in a primary mathematics classroom. *Cambridge Educational Research e-Journal, 11*, 222-235. https://doi.org/10.17863/CAM.114533

Link to the article online: https://www.repository.cam.ac.uk/handle/1810/377844

Published online: December 2024

Cambridge Educational Research e-Journal
2024, VOL. 11
DOI: https://doi.org/placeholder

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Midas syndrome? Commognition as a lens for research on how stem sentences feature in a primary mathematics classroom

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ABSTRACT
This article outlines how commognition - *communication and cognition* - (Sfard, 2008) was utilised as a theoretical lens to guide doctoral research. My doctoral research aims to make sense of stem sentences, which are speaking scaffolds used in primary mathematics classrooms in England. I challenge an oversimplified narrative between mathematical thinking and communication, which endorses stem sentences as faultless discursive objects or *d-objects* – a narrative Dr. Anna Sfard names '*Midas syndrome*' (Sfard, 2019, p. 98). Contrary to popular opinion, stem sentences are not faultless and how they feature in everyday classroom discourse is not well documented. My significant contribution to knowledge is a critical stance towards an endorsed-by-many practice and an evaluation of the utility of theory to analyse audio-visual data from the primary mathematics classroom. To support my argument, I outline an episode of classroom observation which features *commognitive conflict,* where stem sentences also feature, and how a class teacher expertly navigates a learner towards a mathematical realisation through visual mediation. The article is structured using guiding principles which exemplify and evaluate the appropriateness of commognition – gathered from Sfard's seminal work '*Thinking as communicating: Human development, the growth of discourses, and mathematising*' (2008) and related works – and the impact on my doctoral research when observing communication-in-situ in multilingual primary mathematics classrooms.

KEYWORDS
Stem sentences, discursive objects, primary mathematics, commognition, multilingualism

Commognition as a lens for research

Situated between disciplines of mathematics education and sociology, Sfard's (2008) theory of commognition addresses issues of teaching, learning and what it means to be human (Presmeg, 2016, p. 423). Thinking occurs through individualised interpersonal communication *and* collective communication, where we embody the communication of other *mathematists* (those who participate in mathematical discourse). The term commognition accentuates Sfard's principle that interpersonal communication and individual thinking are two sides of the same coin; they are recursive, multimodal and complex (Sfard, 2008, p. xvii).

The notion that communication and cognition are interlinked as commognition has impacted my doctoral research in multiple ways. Firstly, Sfard (2008) expands beyond word-based communication and considers communication on both object and meta levels. *Words* and *visual mediators* are discursive objects *(d-objects)* used to signify primary mathematical objects *(p-objects)*. The use of d-objects to mathematise can also signify endorsed mathematical *narratives* and embed meta-rules within *routines* (see Table 1). Commognition has scaffolded my conceptualisation of stem sentences, which are a non-research driven practice, and how they feature in everyday mathematics discourse as d-objects. On an object level, stem sentences as d-objects have power to promote or diminish meaning alongside other d-objects in the moment. On a meta level, how stem sentences feature in routines will reveal endorsed narratives about the nature of

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commognition.

Table 1
Four categories of commognition summarised (Sfard, 2008)

Type	Category	Definition
Object-level	Words	Mathematical words (usually nouns/objects and then later adjectives) to describe these objects in later lexical discourse. They may also appear in colloquial discourse more discipline may be encouraged to be used around this word.
Object-level	Visual mediators	Visual mediators include primary objects that pre-exist the discourse and artifacts created especially for the sake of communication (e.g., written symbols). These mediators are used in special ways and are part of a parcel of the act of communication and this the thinking process.
Meta-level	Narratives	A narrative that is regarded as reflecting the situation in the world and labelled as true. When the term appears without any mention of the endorser. it is to be understood that the narrative is consensually endorsed by the community of the relevant discourse. In Mathematics, endorsed narratives are those that constitute mathematical theories.
Meta-level	Routines	A set of metarules defining a discursive pattern that repeats itself in certain types of situations.

By applying theory to stem sentences and observing them in everyday classrooms, I hope to contribute a critical analysis, synthesis of research and practice, and an evaluation on the utility of theory to make sense of this non-research driven practice. Previous studies, which have utilised commognition, are extremely varied as they focus on certain aspects of the theory in more depth (Presmeg, 2016). This paper is primarily concerned with stem sentences as d-objects and the narratives they endorse about commognition. To focus my argument, I have designated guiding research principles to shape the article. These guiding principles will structure the paper where, firstly, I outline and critically build upon the existing literature around stem sentences. Secondly, I explore the usefulness of commognition as a theory to conceptualise stem sentences, as well as the impact on the chosen methodology. Finally, guided by my research questions (below), I outline preliminary findings using an episode of learning obtained from observational data.

- **Research Question 1:** How do stem sentences feature in within mathematical discourse about the multiplicative conceptual field in multilingual primary classrooms?
- **Research Question 2:** How do teachers use stem sentences to mediate pupil contributions to mathematical discourse?
- **Research Question 3:** How do teachers navigate commognitive conflict where stem sentences feature?

Review of existing literature

Guiding principle 1: Stem sentences are a non-research driven practice and warrant further attention

Stem sentences are 'fill in the gap sentences' used to describe mathematical objects and connect thinking and communicating. They are normalised in English primary school mathematics classrooms, yet

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2024, VOL. 11
DOI: https://doi.org/placeholder

they are a non-research driven practice and therefore warrant further attention. Stem sentences are often conflated with sentence stems, which are more widely researched. Sentence stems (sentence starters) such as '*I noticed that...*' can be used in mathematics classrooms to draw out learners thinking, begin discussions or support mathematical argumentation (Barclay, 2021, p. 13; Gaunt & Stott, 2018). However, this paper is concerned with *stem sentences*, sometimes referred to as '*language structures*'. Table 2 exemplifies stem sentences and was created from a preliminary documentary analysis of professional development materials published by the National Centre for Excellence in the Teaching of Mathematics (NCETM, 2019).

Table 2
Examples of stem sentences from the National Centre for Excellence in the Teaching of Mathematics (NCETM) professional development materials on multiplication and division, year groups 2-6, (2019)

Type	Fill in the gap structure	Context specific	Generalization
Purpose	To signify relationships in structure	To signify context	To generalise about the rules of mathematics
Examples	'There are ___ equal groups of ___.' 'Ten is double five, so ___ tens is double ___ fives.' '___ multiplied by ten is equal to ___.' '___ is ten times the size of ___.'	'The ___ costs ___ p.' 'There are ___ people in the cinema this evening.'	'Factor times factor is equal to the product.' 'The product is equal to factor times factor.' 'When one of the factors is two, the other factor is half of the product.'

In conjunction with these documents, the Department for Education (DfE) non-statutory guidance in KS1 and KS2 primary mathematics (Morgan, 2020, p. 6) is an additional document where stem sentences are heavily endorsed:

> The development and use of precise and accurate language in mathematics is important, so the guidance includes 'Language focus' features. These provide suggested sentence structures for pupils to use to capture, connect and apply important mathematical ideas. Once pupils have learnt to use a core sentence structure, they should be able to adapt and reason with it to apply their understanding in new context.

The NCETM is a popular professional development and resource provider in England, funded by the DfE with 40 Community Maths Hubs across the country. The non-statutory guidance (DfE, 2020) features a 'Language focus' box which contain stem sentences. The release of the non-statutory guidance (Morgan, 2020) was shortly after the release of professional development materials from the NCETM and features similar language and explicit reference to the use of stem sentences in the classroom.

Even though stem sentences feature and are endorsed by experts in public documents, they are not '*... research-driven (...) but rather a practice that was observed in action in Chinese classrooms as being effective*.' (Coles & Helme, 2022, p. 16). Coles and Helme (2022) are referring to the Shanghai exchange, documented in The Boylan Report (2019, p. 74), which observed teachers modelling '*precise mathematical language in full sentences when explaining and responding to pupils*.' The dearth of systematic and empirical research on teachers' use of stem sentences in the everyday primary classroom in the UK is staggering. Stem sentences seem to be endorsed without scepticism and how they truly feature in the verbal discourse of the primary mathematics classroom is obscure.

Sfard (2008) endorses the narrative that thinking and communicating are iterative, recursive, tantamount to each other and extraordinarily complex. This is in contrast the Midas syndrome narrative endorsed in the limited literature on stem sentences. Coles and Helme (2022) discuss the impact that the NCTEM's professional development materials (2019) have on teacher practice using interviews. Several teachers interviewed are in praise of stem sentences, sometimes referred to as language structures by the teachers. Coles and Helme (2022) themselves acknowledge the multidirectional relationship between thought and communication in the discourse of mathematics. Further to this, interviewed teachers suggest that visual representations are important in conjunction with stem sentences to provide *'structure'* to thinking and communicating (Coles & Helme, 2022, p. 15) and endorse a narrative that stem sentences can *'underpin'* learner's thinking (Coles & Helme, 2022, p. 16).

However, there is also an implicit endorsement of the narrative of *Midas syndrome*, where stem sentences are in danger of promoting a faultless and oversimplified dichotomy between speaking and thinking: that if you are saying it, you think it (Sfard, 2008, 2012, 2019). This is demonstrated when another teacher discusses working with young learners and how '*... you realise that if you give them the language structure then they can reason. If you don't, then what hope have we got...'* (Coles & Helme, 2022, p. 15). I believe that the relationship between stem sentences, as d-objects, and p-objects is more complex and permeated by other aspects of discourse such as visual mediators. Although the teacher accounts outlined in Coles and Helme (2022) provide an interesting discussion, they are in fact anecdotal. Not only are stem sentences a non-research driven practice, with limited existing literature, but how such d-objects *usher* understanding in relation to other modes of communication, and the commognitive narrative they endorse, is still unclear (Sfard, 2012).

Guiding principle 2: Mathematics is unlike any other subject

Due to the multidirectional relationships between signifiers and the signified including concrete, pictorial, symbolic and linguistic representations, mathematics is unlike any other subject and the communicative aspect of mathematics should not be overlooked (Haylock & Cockburn, 1989; Sfard, 2008). Mathematical discourse – *'multimodal communication conducted in accordance with a set of meta-linguistic rules'* (Newton, 2012, p. 69) - is unique, often revolving around elusive mathematical objects which undergo change (Sfard, 2008, p. 128). Underpinned by commognition, I am framing mathematics as its own distinct discourse, as opposed to a register, as a teacher may decide to move between registers, outside of registers and make in the moment choices to maximise learner *mathematising* (Kassim-Lowe, 2022; Prediger et al., 2019; Sfard, 2008). Commognition has been particularly warranted as a framework since it is specifically designed to understand the role of communication in instances of mathematics teaching and learning.

Guiding principle 3: Multiplication represents a large conceptual shift in mathematical thinking

Within the discourse of mathematics, multiplication represents a large conceptual shift in mathematical thinking, linking concepts across primary school mathematics where the language used to represent multiplicative structures can be ambiguous (Harel & Confrey, 1994). I aim to demonstrate the complexity of stem sentences (as d-objects) which describe primary objects (p-objects), which can be felt or seen and how they signify and reify multiplicative structures. The choice to focus on multiplicative structures as p-objects derived from reading the complex scholarly work around the Multiplicative Conceptual Field (MCF). The MCF is multimodal (symbolic, linguistic, concrete, abstract and visual) and represents a huge conceptual shift from thinking additively to multiplicatively (Harel & Confrey, 1994). I will frame stem sentences as d-objects which teachers might utilise to mediate mathematising and support learners in navigating this huge conceptual shift in mathematical thinking. Because the MCF presents such a leap in mathematical thinking, it is likely that *commognitive conflict* may occur - where there is a misalignment between thinking and communicating. Stem sentences are not exempt from this occurrence; not everything they touch turns to gold. To further criticise the Midas syndrome narrative of stem sentences, I identify an instance where commognitive conflict occurs and analyse the expertise of the teacher, who navigates this misalignment between thinking and communication. Commognitive conflict becomes evident during observation when a d-object does not describe a p-object and something may be said to be true even though it is not endorsed by the meta-rules of mathematics (Sfard, 2008).

Guiding principle 4: There is no one d-object which demonstrates 'Midas Syndrome'

In the limited existing literature on stem sentences explored above, there seems to be a *normalised narrative* around the relationship between mathematical thinking and communication: if you say it, you think it. For a practice to be considered *'normal'* it should be evident across the educational community and endorsed by those who are considered community experts (Sfard, 2008). Stem sentences have become a normal aspect of primary mathematics teaching and feature heavily in NCETM and DfE materials, as well as privatised schemes of work such as White Rose Education (WRE). I have framed such documents as *boundary objects* which mobilise stem sentences from written to verbal classroom discourse (Akkerman & Bakker, 2011). When a d-object is non-contested and is believed to be faultless, the narrative it endorses is known as '*Midas syndrome'* (Sfard 2019, p. 98). There is no one d-object which can cement mathematical thinking and teaching mathematics is more multimodal and complex than this. This paper is particularly concerned with research question 3 *'How do teachers navigate commognitive conflict where stem sentences feature?'* which aims to criticise the oversimplified narrative endorsed by boundary objects through demonstrating that where stem sentences feature, commognitive conflict can also feature.

Guiding principle 5: Teachers are interlocutors and pupils are mathematists

Teachers and pupils are *mathematists*; individuals who participate in collective discourse, communicating and thinking with mathematics, each other, and themselves to support realisations (Sfard, 2008). Teachers are also *interlocutors,* who will make multiple in the moment choices and select d-objects to communicate about p-objects to support realisations; language might be planned, spontaneous or influenced by boundary objects (see Table 3).

Table 3
Positionality of Research Participants

	Teacher	**Learner**	**Researcher**
Role	Interlocutor, mediator and mathematist	Mathematist	Observer and mathematist
Definition	A lead mathematists, and 'middleman' negotiating meaning making, mediating discourse, and travelling between multiple sites of meaning.	An individual who participates in and contributes to collective mathematical discourse.	A non-intervening observer of mathematists and interlocutors, who is also passively participating in mathematical discourse.

This paper is concerned with how teachers, as mediators and interlocutors, exist between resisting and promoting the normalisation of the Midas syndrome narrative of stem sentences by analysing how teachers navigate instances of commognitive conflict – where thinking and communication misalign. Despite their widespread endorsement, commognitive conflict may arise where stem sentences feature, casting further doubt on their golden reputation. Sfard (2008) states that commognitive conflict can be interpersonal or intrapersonal and occurs when mathematists are trying to find a commognition in common but there is tension.

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Guiding principle 6: Multilingualism as the norm

I am particularly interested in settings where most of the pupils are multilingual and teachers are more aware of how they, as interlocutors, communicate mathematically. Multilingualism is positioned as the norm in Language in Mathematics Education (LiME) research, of which Sfard's work is included (Phakeng & Moshkovich, 2013; Planas et al., 2021). Multilingual mathematics classroom teachers will be well versed in navigating language dilemmas (Adler, 1999; Prediger et al., 2019). Therefore, such settings are appropriate to further explore research question 3: '*How do teachers navigate commognitive conflict in situations where stem sentences feature?*'. How teachers 'usher' (Sfard, 2012, p. 6) learners in the everyday classroom, moment to moment, and how stem sentences feature, is ambiguous and rarely reported on (Kassim-Lowe, 2022; Moschkovich, 2021). The non-interventionist observational approach I have adopted is in response to Moschkovich's (2021) claim that LiME researchers need to know more about the specific details of classroom interactions to make sense of how teachers navigate communication dilemmas (Planas et al., 2021).

Table 4

The four categories of Commognition Expanded (Sfard, 2008)

Overarching category	Components	Meaning
Words	Passive use	Stage 1: Not actively using a word but responding to use of a word.
	Routine-driven use	Stage 2: Active use of a word restricted by some routines and as a part of constant discursive sequences.
	Phrase driven use	Stage 3: When a word is used with a phrase, associated with that phrase.
	Object driven use	Stage 4: When the object signifies the language.
	Saming	Giving one name to a number of things which are to be considered the same.
	Reifying	When talk about processes is replaced by talk about static narratives.
	Encapsulating	Generalisation about mathematical objects indicated by a potential shift in language.
Visual mediators	Pictures Symbols Concrete Gestures	A visual indicator of a mathematical structure which may be static, animated, drawn or printed, embodied or concrete.
Routines	Deeds	An exchange which happens in a routine which is closed and transactional.
	Rituals	Socially conducive ways of working mathematically in a lesson.
	Explorations	Less transactional and more exploratory use of language to endorse or reject mathematical narratives.
Endorsed narratives	Mathematical Social	Something labelled as being social or mathematically true in the contexts of mathematics teaching and learning. A narrative believed to be untrue is rejected.

Cambridge Educational Research e-Journal
2024, VOL. 11
DOI: https://doi.org/placeholder

Guiding principle 7: Commognition is a useful lens for conceptualising stem sentences as d-objects

As mathematists, teachers and learners engage in discourse to make sense of primary objects (p-objects) which are any perceptually accessible entity that we can see, hear, and touch in mathematics. The use of discursive objects (d-objects), such as stem sentences, can endogenously or exogenously expand or diminish mathematical meaning-making for an individual or collective. Sfard (2008, p. 4) suggests that one of the main roles of d-objects is to act as the *signifiers* of p-objects, the utilisation of which has the potential to create *moments of realisation.* Sfard's (2008) holistic commognitive approach considers discourse as encapsulated not only verbally but *spatially* with visual mediators (Dickson et al., 1984; Sfard, 2008). Sfard (2008) rejects the dualist separation of spatial and verbal competencies and instead considers the two components as an inseparable part of multimodal discursivity that occurs during mathematising. The relationship between the verbal and spatial in everyday classroom discourse is not always clear cut and warrants negotiation, probing and processing as teachers decide which d-objects they will employ to maximise sense-making of p-objects (Newton, 2012; Sfard, 2008). To unpack the complex relationship between stem sentences and other d-objects, I have used Sfard's library of work to expand further on the original four categories which will be used to analyse instances of commognitive conflict (see Table 4). It was necessary to further expand these categories after my initial visits to classrooms where the complex relationship between thought and communication became apparent.

Since Sfard's work could be criticised for its highly theoretical nature and lack of accessibility, I aim to contribute a better understanding of not only the complexity of stem sentences but commognition itself through an explicit analysis of real-life classroom data.

Methodology

Ethics, sampling and consent

I have used opportunistic and purposive sampling to ethically gain access to schools and classrooms via a gatekeeper. The gatekeeper was a mathematics lead with an interest in research. Each teacher, pupil and parent/carer involved was asked to read a privacy notice, information sheet and consent form before giving written consent. At appropriate intervals, participants were reminded of the right to withdraw, and anonymity was ensured. Both parents and children were invited to sign consent forms which gave permission for audio-visual materials to be transcribed. Consenting children wore green stickers during the lesson. It was considered unethical to withdraw non-consenting children from the lesson so only children who gave consent had their audio data transcribed which was pseudonymised. All the people involved were given culturally sensitive pseudonyms.

Audiovisual observations and interviews

Sfard's (2008) framework has urged me to acknowledge myself as both a researcher, observer *and* a participant in the learning context (see Table 2). I have considered any communication that directly or indirectly involves my presence by transcribing audio data verbatim. I obtained both audio and visual data because Sfard's (2008) commognitive framework explicitly notes that *it is not just words which make up discourse but visual mediators.* Therefore, use of a recording camera was warranted to capture the whole picture of how discourse sounds and *looks*. Rewatchable audio-visual methods captured what is not noticed in the classroom (Ching et al., 2019). Post-observation semi-structured teacher interviews were conducted and transcribed verbatim where teachers were encouraged to make sense of specific episodes in the lessons to support the analysis (Sfard, 2008).

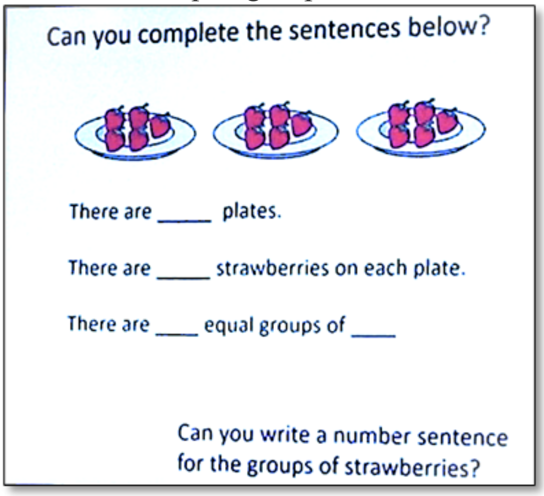
Preliminary findings

Lesson background

The episode took place in a year 3 (ages 7-8) multilingual mathematics classroom. After teaching long term in year 6, the class teacher had moved to a year 3 classroom. The episode was recorded during whole class input and involves the utterances of the class teacher (pseudonym: Charlie) and two learners (pseudonyms: Nasir and Fatima). The lesson was centered around the concept of ‘equal groups’, where stem sentences featured heavily as part of the mathematical discourse. Primarily, stem sentences were used to describe pictorial representations of equal group structures, moving from the most context specific, fill in the gap style structures, to a more generalised phrase (see Figure 1). The stem sentences appeared on the interactive whiteboard and were taken from White Rose Education materials.

Firstly, the groups are signified (multiplier – ‘There are 3 plates’) and then the number of items in each group (multiplicand – ‘There are 5 strawberries on each plate’). The final sentence uses the word ‘equal’ – ‘There are 3 equal groups of 5’. The teacher asked a few children to read the sentences, inserting the correct numbers and spoke about which number represented the group, and which number represented the amount in each group. Charlie did not expect every child to use the scaffolded sentence structure but did use the stem sentences (d-objects) to highlight the equal group structure (p-object).

Figure 1
Stem sentences, Equal groups



When Charlie asked what this pictorial and linguistic representation would look like as a symbolic representation (number sentence), commognitive conflict occurred, where a learner made an utterance which is mathematically untrue - ‘*three equals five’*. (See Transcription 1 below and Figure 2: A visual representation of the order of discourse).

Transcription 1
‘Three equals five’

Charlie (Class teacher): What would we do if we were writing a number sentence then? What would we do? We’ve got the...we’ve got the plates. We’ve got **three** plates, so that’s going to be in there, isn’t it? We’ve got **five** strawberries on each plate so that’s going to be in there isn’t it? So, what do we write. (...) Nasir?
Nasir: Three equals five.
Charlie: (writes 3 = 5 on the whiteboard) Three equals five? Does that look right? Cos three and five aren’t the same, are they? **(gesture hands to show imbalance).** What do you think, Fatima?
Fatima: Five times three?
Charlie: So, do we have five groups of three? Five groups of three **(writes 5 x 3)** Have we? Three groups of five? **(writes 3x5)** Yeah? What’s the answer? (children shout fifteen intermittently).

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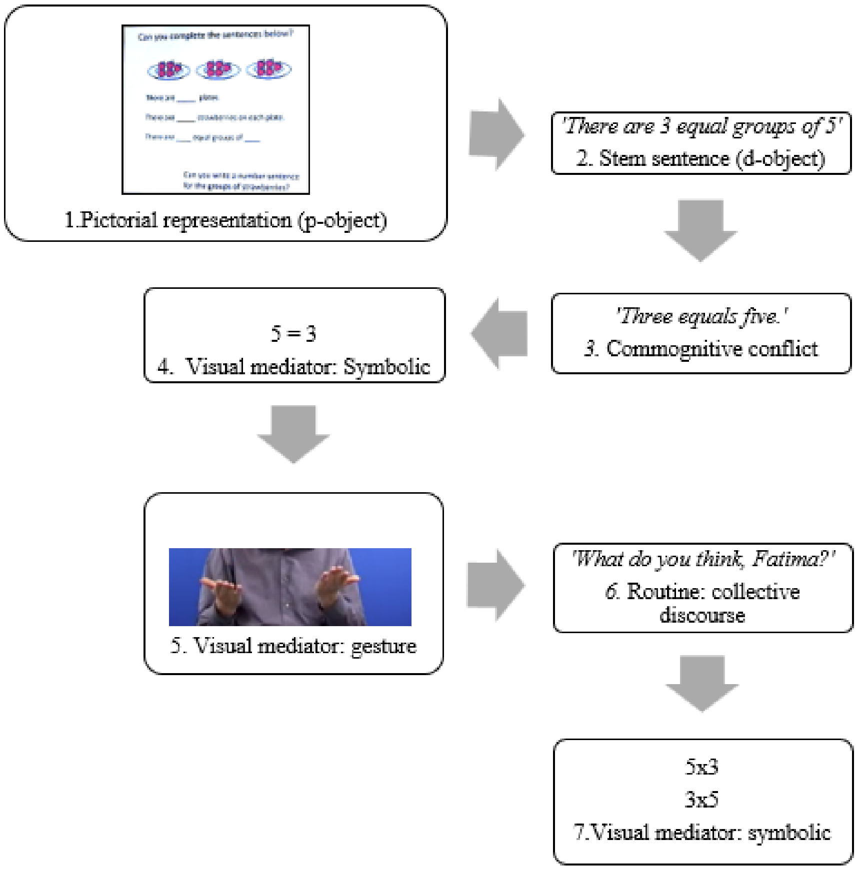
DOI: https://doi.org/placeholder

Figure 2
Visual representation of the order of discourse



Words and routines

Initially in the lesson, stem sentences, as d-objects, feature in a *routine-driven use,* whereby the learners engaged in a ritual of inserting the correct number into the gaps and repeating the sentence multiple times. This then changed to a *phrase-driven use,* whereby the class teacher asked the children to move from a linguistic representation to a symbolic representation based on the phrases uttered (see Table 5).

Table 5
Commognition as a lens to view words and routines

Overarching category	Components	Feature
Words	Routine-driven use	Stage 2: Active use of a word restricted by some routines and as a part of constant discursive sequences by repeating same sentence.
	Phrase driven use	Stage 3: When a word is used with a phrase, associated with that phrase and then associated with consequent utterances *'Three equals five'*.
	Saming	'=' use of the equals sign meaning 'the same as'.
	Reifying	*'There are 3 equal groups of 5'*. Static narrative of pictorial representation of multiplicative structure.
Routines	Rituals	Multiple learners had the chance to fill in the gaps and repeat the sentence *'There are 3 equal groups of 5'*.

Nasir associated the word ‘equal’ with the stem sentences and this featured in his utterance ‘*three equals five’*. In an interview with the class teacher (see Transcription 2), Charlie, we discussed why Nasir made this utterance:

Transcription 2
‘And saw the word equals.’

> **Charlie:** (...)I think that’s what he thought. He got the number on each plate, and he got the number of plates.
> **Researcher:** And then saw the word equals?
> **Charlie:** And saw the word equals.

Charlie did not think the child truly believed that ‘*three equals five’,* but rather that it was the correct thing to say in the moment given the stem sentence highlighted the words *‘three’*, *‘five’* and *‘equal’*. Instead, when a second child uttered ‘*five times three’,* the class teacher reiterated the phrase ‘groups of’ when writing the multiplication symbol, where *‘5 groups of 3’* was written as 5x3 and *‘3 groups of 5’* written as 3x5 implying that the pictorial representation was more likely to be represented by 3x5 which Charlie thought of as *‘3 groups of 5’.*

Sfard (2008) discusses the use of d-objects to reify a p-object. This is where an object previously talked about in terms of processes is now talked about in terms of its static narrative, e.g. *‘There are 3 groups of 5’.* In this case, the class teacher was starting from a pictorial representation of an equal group structure, which was given the status of a p-object. Stem sentences were used to describe this static narrative and therefore reify the object. Additionally, saming occurred in the episode as well. In the first instance the teacher used visual mediators to gently prove that three and five aren’t the same and support the child through their commognitive conflict. A generalisation was made about the equals sign, as a p-object, when the teacher said ‘*Cos three and five aren’t the same, are they?’.* Although words were important to support realisations about the meaning of mathematical symbols (x – *‘groups of’* and = - *‘the same’*), the role of visual mediators, in this case symbolic and gestural, is essential in navigating commognitive conflict and supporting realisations (see Table 6).

Visual mediators and endorsed narratives

Table 6
Commognition as a lens to view visual mediators and endorsed narratives

Overarching category	Components	Feature
Visual mediators	Pictures	Pictorial representation.
	Symbols	Symbolic representation: 5=3, 5x3, 3x5.
	Gestures	Imbalance gesture.
Endorsed narratives	Mathematical	= ‘the same as’, X ‘groups of’. Gently rejecting narrative that 3=5.
	Social	Endorsing collective commognition. Rejection of Midas Syndrome.

It is important to note that when ‘*three equals five’* was uttered, Charlie did not return to the stem sentence but rather in another direction towards utilising other d-objects: symbolic and gestural visual

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2024, VOL. 11
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mediators. Charlie made connections with the class's collective commognition. Rather than returning to the stem sentence, Charlie opted to write *'three equals five'* as '3 = 5' in its symbolic form and used a gesture of imbalance. During the interview, Charlie stated that this gentle rejection of Nasir's false mathematical narrative gave Nasir time to come to terms with their commognitive conflict (see Transcription 3).

Transcription 3
'If I can draw up what they've said…'

> **Researcher:** But there was just this moment where you had to do something for him to realise. I think for me that something was writing it down cos to see it it's like okay that looks weird.
> **Charlie:** It's difficult isn't it cos sometimes you don't know what their thinking and where the misconceptions come from. So, all you can really do is draw it out. Just say 'This is what you just said' erm...
> **Researcher:** And you are very good at that, actually. You never just say 'no'. You never ever do that.
> **Charlie:** No!
> **Researcher:** (...)Why is that?
> **Charlie:** Just because I know that their misconceptions are something I can't I mean obviously clearly, I can't even think of it now but like I can't like and I don't always know why that misconception is there.
> **Researcher:** Yeah.
> **Charlie:** So, I just give them a little bit of room to try and get it right in their own head.
> **Researcher:** Yeah.
> **Charlie:** If I can draw up what they've said. Sometimes when you see it on the board. You realise the mistake and that it doesn't make sense. So you get a chance to think again. So, I do try and give them the opportunity to think about what they just said and then also give the others some input.

For Charlie, who utilised their whiteboard throughout the lesson, it was important for the children to 'see' what they had said to make sense of it. Writing or drawing as a medium to express thinking is less fleeting than speech and therefore more time can be taken to give the learner *'a chance to think again'*. This links to Sfard's claim that verbal and spatial competencies should be considered in conjunction (Dickson et al., 1984; Sfard, 2008) and supports the argument around the nature of mathematics as a distinct and complex discourse where no one d-object can generate mathematical realisations. In addition to writing, Charlie used an unplanned gesture of imbalance to gently usher Nasir towards a realisation (see Transcription 4).

Transcription 4
'They're not balanced.'

> **Researcher:** And you do use your hands a lot. I don't know if you know that?
> **Charlie:** No, do I? (laughs)
> **Researcher:** But you did this (imbalance gesture). That's not quite right.
> **Charlie:** Yeah. Ok.
> **Researcher:** And when you did this (imbalance gesture), to me it looked like balance and a scale which made me think about your experience in year 6 and the algebra and that idea of balance. I am wondering if I have read too much into that or do...?
> **Charlie:** No, I probably I mean, yeah. I would say that 'They're not balanced' erm...
> **Researcher:** Although you didn't say that, but you did say it with your hands.
> **Charlie:** With the hands – yeah. I didn't realise that I did that to be honest.

As an interlocutor, Charlie used their own prior experience from teaching and mathematising in year 6 to imply 'imbalance' through a gesture when the utterance *'three equals five'* was made by Nasir. Although Charlie uttered '*Cos three and five aren't the same, are they?'* whilst gesturing imbalance, they stated that they would usually say '*They are not balanced*'. Although the word balance itself was not uttered it was implied and communicated through a gesture. A gesture which the teacher claims they didn't realise they made and was not planned. Further to this, Charlie indicated that, when commognitive conflict arises, they try and give learners '*... the opportunity to think about what they just said and then also give others some input'.* Sfard's (2008) conceptualisation of commognition as both part of intrapersonal individual discourse and interpersonal collective discourse reiterates the importance of a collective endeavor towards mathematical sense-making. Because of the complex nature of thinking and communicating in mathematics, not all communication can be planned. Some communication will be spontaneous and responsive to the learners' commognition; to usher realization or to navigate commognitive conflict. These in the moment decisions are a result of teacher expertise and their own skills, as an interlocutor and mathematist, whilst drawing on the thinking of others.

This episode of learning does not endorse the narrative that 'if you say it, you think it' on multiple levels. Firstly, the class teacher did not think the child truly believed that *'three equals five'* but rather spliced together key words from using the stem sentences. Secondly, when translating from one d-object to another - stem sentence to a symbolic representation of the pictorial representation (which was given the status of a p-object) - commognitive conflict occurred. When this commognitive conflict occurred, Charlie did not return to the stem sentence but used gestural and symbolic visual mediators to support the mathematical realisation that three cannot equal five and endorse a mathematical narrative around the meaning of both the equals and multiplication symbols.

Conclusion

In this early stage of my doctoral research, I have aimed to demonstrate how I have utilised Sfard's scholarly work to conceptualise the role of stem sentences as d-objects and outline an approach to observational data analysis. I aim to provide an example of how communication and thinking are recursive and complex. I also demonstrate how theory can be used to understand the complexities of practice and practice can be used to exemplify aspects of theory which may be criticized for being inaccessible. In my aim to demystify the Midas syndrome narrative of stem sentences, it is also important to note that I am not arguing that stem sentences *cause* commognitive conflict. Instead, I have exemplified a theorised view of stem sentences as d-objects, which are not faultless and that should be considered in conjunction with other visual mediators, such as gestures – some of which are not planned but are an in the moment reaction to student's thinking. Teachers as mathematists themselves and interlocuters, who move between multiple discourses and sites of meaning, have multiple communicative faculties and a wealth of expertise at their disposal. More classroom-based research is needed to explore the complexities of communicating and thinking where stem sentences feature since they are a normalised, endorsed-by-many practice. An over-simplified view of stem sentences discredits the wisdom and adaptability of teachers when mediating mathematists in the moment (Sfard, 2019).

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